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m_{ij} = migration rate of infected individuals from jth region to ith region.

With these assumptions, we have n_{ii} m_{ii} 0. Then, for i = 1,..., p,

If the system (2) is well-posed, we can find a bound for the population of susceptibles in each patch. We can then extend the same idea to obtain the linear system of I-equations only. (See Appendix for details.)

From Theorem 1 in the Appendix, we have

Thus, the total population of susceptible individuals in the ith region is limited. So we can write

$_i$ = β× (total population in the ith region without infection)

$d_i = {}_i + {}_i\gamma$

where d_i represents the total death rate in the ith region. Thus, for i = 1,..., p, the equations of the I-only model are

This can be rewritten as

where $\mathbf{I}(t) = (I_1(t), I_2(t),..., I_p(t))^T$ and

two regions to some extent. For example, for Case 2, denoting

calculation shows that

From this, one can derive conditions for the travel rate of susceptible individuals which can either help a disease which is otherwise dying out to persist locally or cause an otherwise partially persistent disease to persist globally in the two regions (see Appendix for the details). It follows th-6.19e trav999999(1)-5.se

condoms, drugs and travel restrictions. The costs that can

Figure 4

Reduction in transmission. The effect on the eradication threshold condition T_0 of reducing the transmission rate over a twenty year period to two fifths of the current value by decreasing the infection rate from 2010 levels to two fifths its current value, in 2030. A. Assuming population demographics remain unchanged. B. Assuming 3% population growth per year. Populaabstinence and monogamy [68]. Travel/immigration restrictions will reduce m_{ij} and m_{ji}, but we do not expect that these will be realistic; past attempts to restrict movement based on travel or immigration have simply driven the epidemic underground [69].

Note that we assume a population growth of 3% per year. This corresponds to the maximum population growth rate reported by the CIA World Factbook. We use the maximum growth rate so that cost estimates give an upper bound of what may be realistic for donor spending. We also assume that HIV-positive cases will continue to increase at a rate of 3% per year, if no interventions are undertaken.

Our cost formula is thus

$$C(n,m,r) \quad 3 \quad 10 \quad \frac{1.03^{r} \quad 1}{0.03} \quad 100 \quad 0.02 \quad n$$

$$3.3 \quad 10 \quad \mathrm{i}\,8\,8^{r}9\,2\,8\,2\,0\,0\,0\,7\,3\,2 \quad \mathrm{T}$$

.

where C is the cumulative cost, n is the proportion of men who must ultimately receive condoms, m is the fraction of infected individuals who receive treatment, r is the timescale, ϵ is the (fixed) cost of distribution and education, and is the cost of treatment (including the cost of paid health workers, testing etc). Thus, in the first term of (17), there are 3 billion men, whose numbers increase by 3% every year, given 100 condoms each, at a cost of \$0.02, with a fixed condom education campaign of \$60 million. In the second term of (17), we assume that one tenth of infected individuals require treatment; thus, currently there are 3.3 million individuals requiring treatment, whose numbers increase by 3% every year, at an average cost of \$2500 per patient per year.

If we provided condoms to 3/5 of all men over a period of twenty years and treated nobody, then the cumulative cost is

Thus, by steadily reducing the infection rate each year, the costs would blow up, way beyond the available funds and we would not even reach the eradication threshold. See Figure 5A.

If we treated everyone who requires treatment for twenty years, then the cumulative cost is

assuming no fixed costs of condom education.

If instead we aimed to treat 50% of people requiring treatment and provided condoms to 1/5 of all men within twenty years, then the cumulative cost is

Thus, treatment is significantly more expensive than prevention (as expected).

Conversely, suppose we aimed to reduce infection to two fifths of its current value and treat everybody infected over the next five years. Then the cumulative cost is

Thus, we could reach our eradication threshold within five years, provide treatment to everyone who needs it and stay within our \$60 billion budget (plus interest). See Figure 5B.

Discussion

We have provided a method for predicting treatment and prevention levels necessary to eradicate HIV/AIDS, based on population and immigration data at the continental level. This method is easily applicable to a finer scale, such as country-level data. The mathematical model is linear, which allows it to be easily generalised. We have also developed a formula to estimate the cost of some of these interventions.

It should be stressed that our model does not attempt to quantify the prevalence of the disease or the time course of infection. The model is a predictor of eradication only and thus should not be used in other contexts. However, it provides us with insight into the degree of intervention

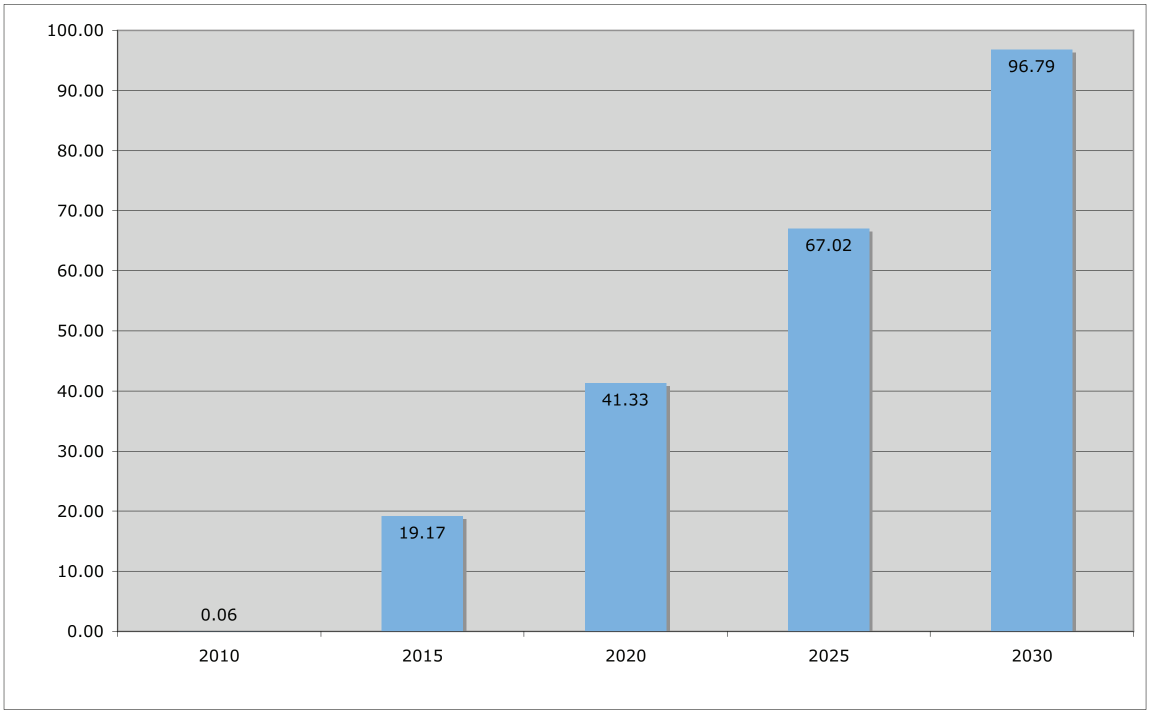

Figure 5

Cumulative cost. A. The cumulative cost of reducing the infection rate to two fifths of its current rate over the next five years and treating everybody who requires it. B. The cumulative cost of reducing the infection rate to two fifths of its current rate over the next twenty years (and treating nobody).

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